

Walter Halls Primary School



MATHEMATICS

KS1 & 2 Calculation

Policy and Vocabulary

Progression

Calculation policy.

The progression of skills within the 2014 KS1 National Curriculum is focused on children moving from concrete (using manipulatives), to pictorial (drawing representations in books), to abstract (using written methods to solve problems).

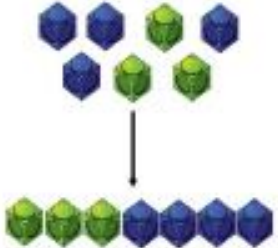
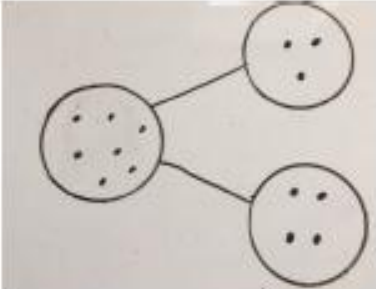
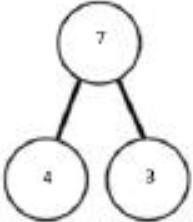
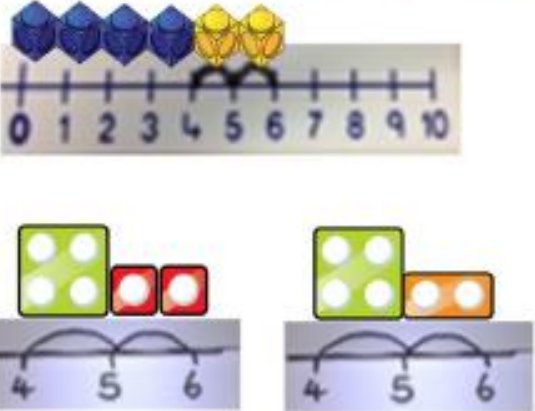
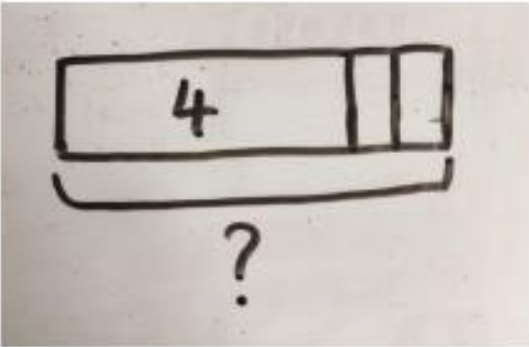

As a wide range of concrete and pictorial representations could be used, a KS1 calculation policy cannot be exhaustive and as a result is a suggested progression from concrete to abstract. EYFS and Lower KS2 have been included for reference.

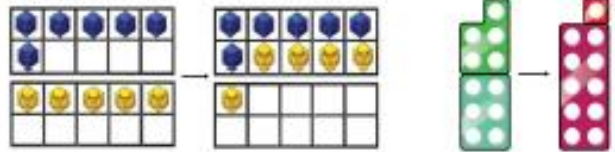
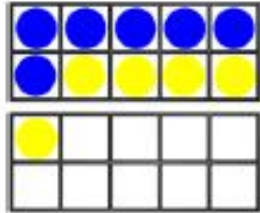

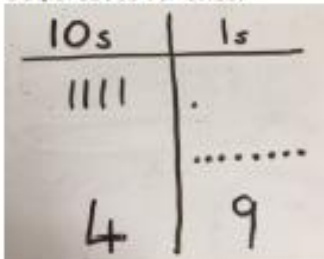
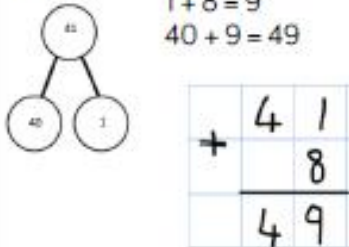
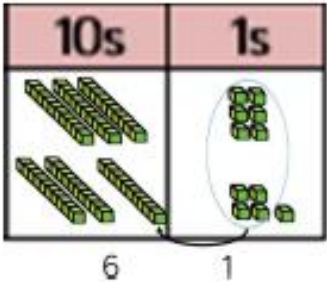
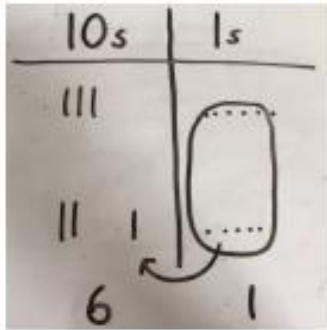
The KS1 policy has been derived from the White Rose example. This allows teachers to make the best use of the planning resources, activities and assessments produced by White Rose.

	EYFS / Year 1	Year 2	Year 3
Addition	<p>Combining two parts to make a whole: part whole model.</p> <p>Starting at the bigger number and counting on- using cubes.</p> <p>Regrouping to make 10 using ten frame.</p>	<p>Adding three single digits.</p> <p>Use of base 10 to combine two numbers.</p>	<p>Column method- regrouping.</p> <p>Using place value counters (up to 3 digits).</p>
Subtraction	<p>Taking away ones</p> <p>Counting back</p> <p>Find the difference</p> <p>Part whole model</p> <p>Make 10 using the ten frame</p>	<p>Counting back</p> <p>Find the difference</p> <p>Part whole model</p> <p>Make 10</p> <p>Use of base 10</p>	<p>Column method with regrouping. (up to 3 digits using place value counters)</p>
Multiplication	<p>Recognising and making equal groups.</p> <p>Doubling</p> <p>Counting in multiples use cubes, Numicon and other objects in the classroom</p>	<p>Arrays- showing commutative multiplication</p>	<p>Arrays</p> <p>$2d \times 1d$ using base 10</p>
Division	<p>Sharing objects into groups</p> <p>Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups?</p> <p>Use cubes and draw round 3 cubes at a time.</p>	<p>Division as grouping</p> <p>Division within arrays- linking to multiplication</p> <p>Repeated subtraction</p>	<p>Division with a remainder- using lollipop sticks, times tables facts and repeated subtraction.</p> <p>$2d$ divided by $1d$ using base 10 or place value counters</p>

Addition

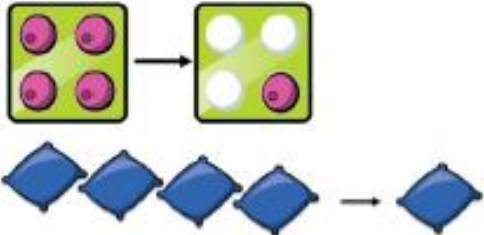
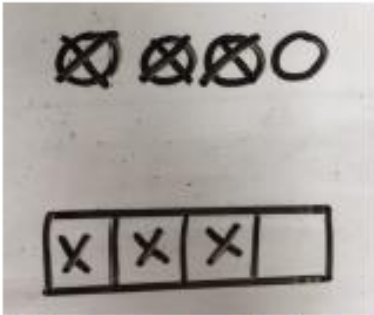


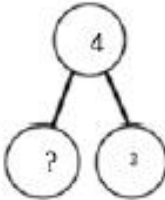

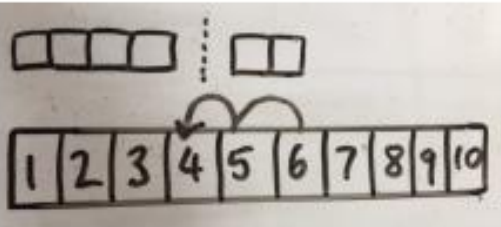

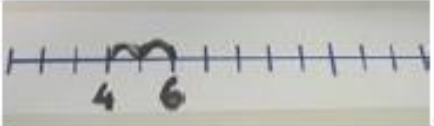
Key Language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' & 'is the same as'.

Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).</p> 	<p>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p> 	<p>$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven.</p> 
<p>Counting on using number lines using cubes or Numicon.</p> 	<p>A bar model which encourages the children to count on, rather than count all.</p> 	<p>The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2$</p> 

Concrete	Pictorial	Abstract
<p>Regrouping to make 10; using ten frames and counters/cubes or using Numicon.</p> <p>6 + 5</p> 	<p>Children to draw the ten frame and counters/cubes.</p> 	<p>Children to develop an understanding of equality e.g.</p> $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$
<p>TO + O using base 10. Continue to develop understanding of partitioning and place value.</p> <p>41 + 8</p> 	<p>Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.</p> 	<p>41 + 8</p> <p>1 + 8 = 9 40 + 9 = 49</p> 
<p>TO + TO using base 10. Continue to develop understanding of partitioning and place value.</p> <p>36 + 25</p> 	<p>Children to represent the base 10 in a place value chart.</p> 	<p>Looking for ways to make 10.</p> <p>36 + 25 =</p> <p>30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61</p> <p>1 5 36</p> <p>Formal method:</p> $\begin{array}{r} +25 \\ 36 \\ \hline 61 \\ 1 \end{array}$

Subtraction

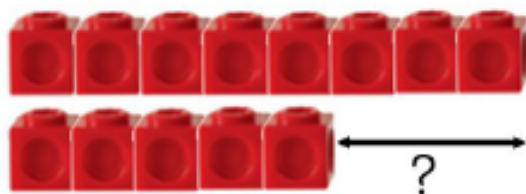
Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Concrete	Pictorial	Abstract
<p>Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).</p> <p>$4 - 3 = 1$</p> 	<p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p> 	<p>$4 - 3 =$</p> <p> $= 4 - 3$</p>  
<p>Counting back (using number lines or number tracks) children start with 6 and count back 2.</p> <p>$6 - 2 = 4$</p> 	<p>Children to represent what they see pictorially e.g.</p> 	<p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line</p>  

Concrete

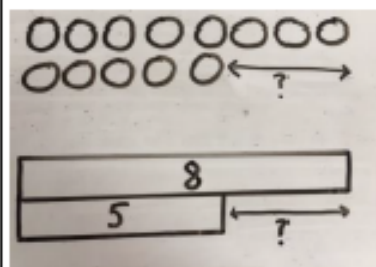
Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5.



Pictorial

Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.



Abstract

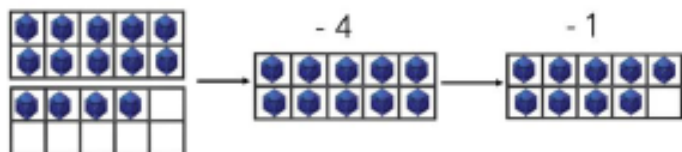
Find the difference between 8 and 5.

8 - 5, the difference is

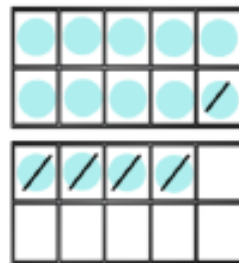
Children to explore why
 $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.

Making 10 using ten frames.

$14 - 5$



Children to present the ten frame pictorially and discuss what they did to make 10.



Children to show how they can make 10 by partitioning the subtrahend.

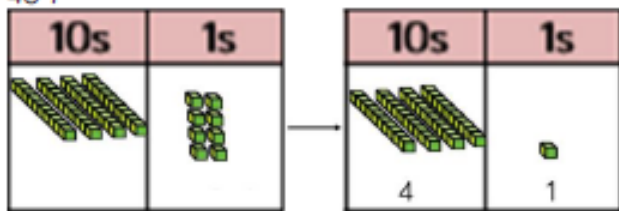
$$14 - 5 = 9$$

$$14 - 4 = 10$$

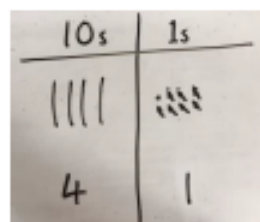
$$10 - 1 = 9$$

Column method using base 10.

$48 - 7$



Children to represent the base 10 pictorially.

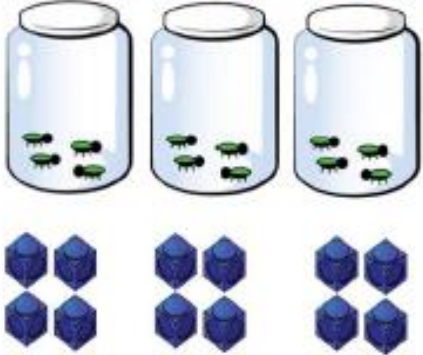
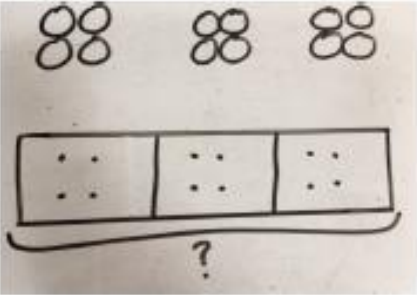
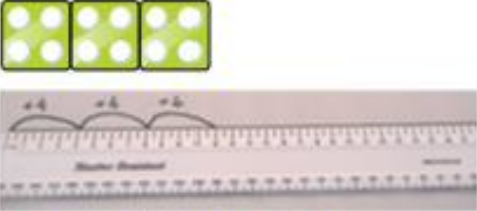
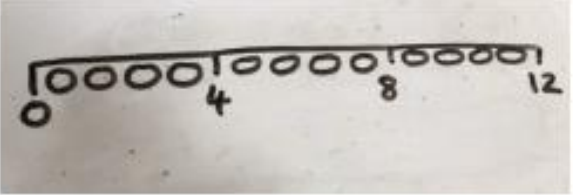
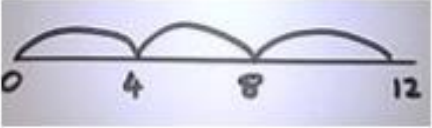


Column method or children could count back 7.

	4	8
-		7
	4	1

Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Concrete	Pictorial	Abstract
<p>Repeated grouping/repeated addition 3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group.</p> 	<p>Children to represent the practical resources in a picture and use a bar model.</p> 	<p>$3 \times 4 = 12$ $4 + 4 + 4 = 12$</p>
<p>Number lines to show repeated groups- 3×4</p>  <p>Cuisenaire rods can be used too.</p>	<p>Represent this pictorially alongside a number line e.g.:</p> 	<p>Abstract number line showing three jumps of four.</p> <p>$3 \times 4 = 12$</p> 

Concrete

Pictorial

Abstract

Use arrays to illustrate commutativity counters and other objects can also be used.

$$2 \times 5 = 5 \times 2$$

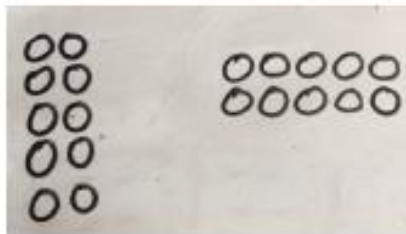


2 lots of 5



5 lots of 2

Children to represent the arrays pictorially.



Children to be able to use an array to write a range of calculations e.g.

$$10 = 2 \times 5$$

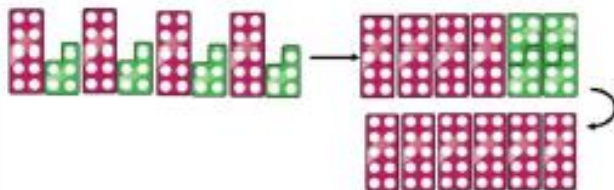
$$5 \times 2 = 10$$

$$2 + 2 + 2 + 2 + 2 = 10$$

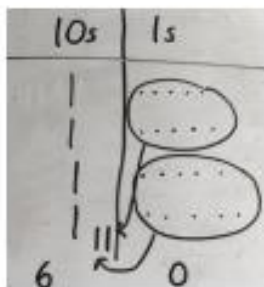
$$10 = 5 + 5$$

Partition to multiply using Numicon, base 10 or Cuisenaire rods.

$$4 \times 15$$



Children to represent the concrete manipulatives pictorially.



Children to be encouraged to show the steps they have taken.

$$4 \times 15$$

$$10 \quad 5$$

$$10 \times 4 = 40$$

$$5 \times 4 = 20$$

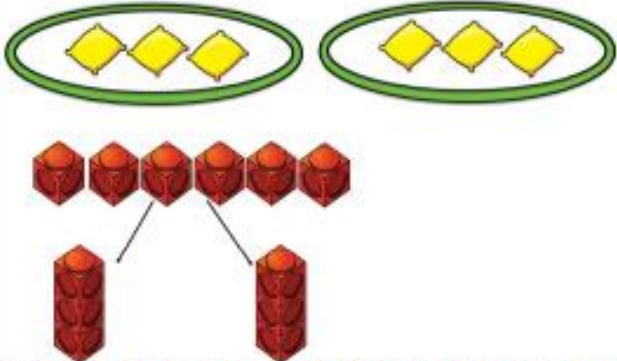
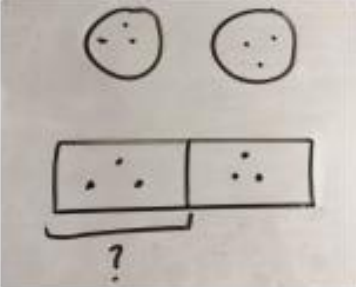
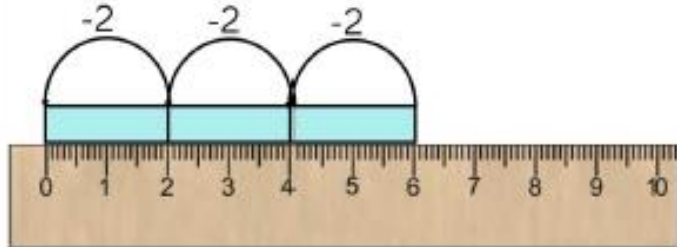
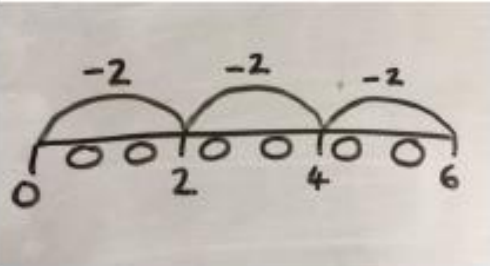
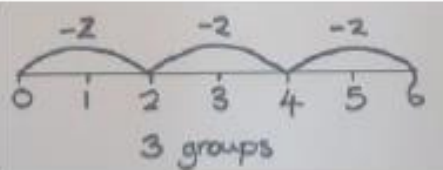
$$40 + 20 = 60$$

A number line can also be used



Division

Key language: share, group, divide, divided by, half.

Concrete	Pictorial	Abstract		
<p>Sharing using a range of objects. $6 \div 2$</p>  <p>The diagram shows two green ovals, each containing three yellow diamonds. Below them is a row of six red cubes. Two lines connect the first and second cubes to a single red cube below, and another two lines connect the third and fourth cubes to another single red cube below, illustrating the process of sharing 6 cubes into 2 groups of 2.</p>	<p>Represent the sharing pictorially.</p>  <p>The diagram shows two hand-drawn circles, each with three dots inside. Below them is a hand-drawn rectangle divided into two equal halves, with three dots in each half. A bracket under the first half has a question mark below it, suggesting a problem to be solved.</p>	<p>$6 \div 2 = 3$</p> <table border="1" data-bbox="1480 518 1883 582"><tr><td>3</td><td>3</td></tr></table> <p>Children should also be encouraged to use their 2 times tables facts.</p>	3	3
3	3			
<p>Repeated subtraction using Cuisenaire rods above a ruler. $6 \div 2$</p>  <p>The diagram shows a ruler from 0 to 10. Three light blue Cuisenaire rods are placed above the ruler, each spanning from 0 to 2, 2 to 4, and 4 to 6. Each rod has '-2' written above it. Below the ruler, the text '3 groups of 2' is written.</p>	<p>Children to represent repeated subtraction pictorially.</p>  <p>The diagram shows a hand-drawn number line from 0 to 6 with circles at each integer. Three arcs are drawn above the line, each starting at an even number and ending at the next even number: from 0 to 2, 2 to 4, and 4 to 6. Each arc has '-2' written above it.</p>	<p>Abstract number line to represent the equal groups that have been subtracted.</p>  <p>The diagram shows a hand-drawn number line from 0 to 6 with circles at each integer. Three arcs are drawn above the line, each starting at an even number and ending at the next even number: from 0 to 2, 2 to 4, and 4 to 6. Each arc has '-2' written above it. Below the number line, the text '3 groups' is written.</p>		

Concrete

Pictorial

Abstract

2d + 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.

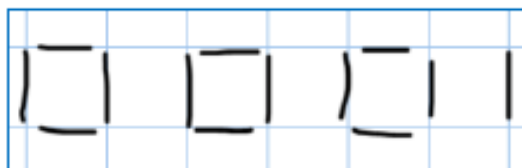
$$13 \div 4$$

Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.

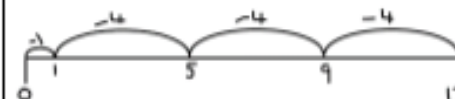


There are 3 whole squares, with 1 left over.

$$13 \div 4 = 3 \text{ remainder } 1$$

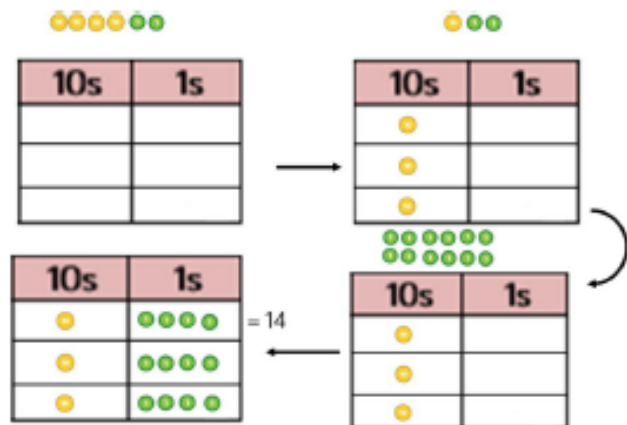
Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

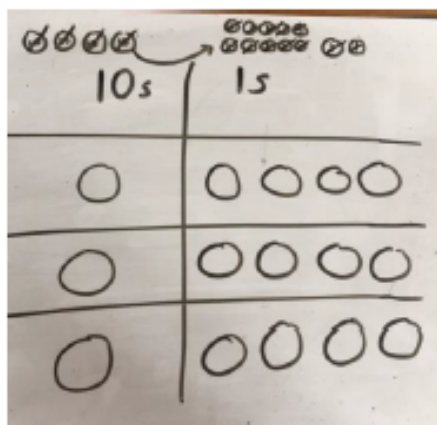


Sharing using place value counters.

$$42 \div 3 = 14$$



Children to represent the place value counters pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

$$\begin{aligned} 42 \div 3 \\ 42 &= 30 + 12 \\ 30 \div 3 &= 10 \\ 12 \div 3 &= 4 \\ 10 + 4 &= 14 \end{aligned}$$

Maths vocabulary for year 1

Number and place value	Addition and subtraction	Multiplication and division	Measure	Geometry (position and direction)	Geometry (properties of shape)	Fractions	General/problem solving
Number	Number bonds, number line	Odd, even	Full, half full, empty	Position	Group, sort	Whole	Listen, join in
Zero, one, two, three to twenty, and beyond	Add, more, plus, make, sum, total, altogether	Count in twos, threes, fives	Holds	Over, under, underneath, above, below, top, bottom, side	Cube, cuboid, pyramid, sphere, cone, cylinder, circle, triangle, square	Equal parts, four equal parts	Say, think, imagine, remember
None	Inverse	Count in tens (forwards from/backwards from)	Container	on, in, outside, inside	Shape	One half, two halves	Start from, start with, start at
Count (on/up/to/from/down)	Double, near double	How many times?	Weigh, weighs, balances	around, in front, behind	Flat, curved, straight, round	A quarter, two quarters	Look at, point to
Before, after	Half, halve	Lots of, groups of	Heavy, heavier, heaviest, light, lighter, lightest	Front, back	Hollow, solid		Put, place, fit
More, less, many, few, fewer, least, fewest, smallest, greater, lesser	Equals, is the same as (including equals sign)	Once, twice, three times, five times	Scales	Before, after	Corner (point, pointed)		Arrange, rearrange
Equal to, the same as	Difference between	Multiple of, times, multiply, multiply by	Time	Beside, next to, Opposite	Face, side, edge		Change, change over
Odd, even	How many more to make..?, how	Repeated addition	Days of the week: Monday, Tuesday, etc.	Apart	Make, build, draw		Split, separate
			Seasons: spring, summer, autumn, winter	Between, middle, edge, centre			Carry on, continue, repeat, what comes next?
			Day, week, month, year, weekend	Corner			Find, choose, collect, use, make, build
			Birthday, holiday				
			Morning, afternoon, evening,				

Pair	many more is...than...?, how much more is..?	Array, row, column	night, midnight	Direction			Tell me, describe, pick out, talk about, explain, show me
Units, ones, tens		Double, halve	Bedtime, dinnertime, playtime	Journey			
Ten more/less	Subtract, take away, minus	Share, share equally	Today, yesterday, tomorrow	Left, right, up, down, forwards, backwards, sideways			Read, write, record, trace, copy, complete, finish, end
Digit		Group in pairs, threes, etc.	Before, after	Across			
Numeral	How many fewer is...than...?, how much less is..?	Equal groups of	Next, last	Close, far, near			Fill in, shade, colour, tick, cross, draw, draw a line between, join (up), ring, arrow
Figure(s)		Divide, divided by, left, left over	Now, soon, early, late	Along, through			
Compare			Quick, quicker, quickest, quickly, fast, faster, fastest, slow, slower, slowest, slowly	To, from, towards, away from			Cost
(In) order/a different order			Old, older, oldest, new, newer, newest	Movement			Count, work out, answer, check same
Size			Takes longer, takes less time	Slide, roll, turn, whole turn, half turn			number(s)/different number(s)/missing number(s)
Value			Hour, o'clock, half past	Stretch, bend			Number facts, number line, number track, number square, number cards
Between, halfway between			Clock, watch, hands				Abacus, counters, cubes, blocks, rods, die, dice, dominoes, pegs, peg board
Above, below			How long ago?, how long will it be to...?, how long will it take to...?, how often?				Same way, different
			Always, never, often, sometimes, usually				
			Once, twice				
			First, second, third, etc.				
			Estimate, close to, about the				

			<p>same as, just over, just under</p> <p>Too many, too few, not enough, enough</p> <p>Length, width, height, depth</p> <p>Long, longer, longest, short, shorter shortest, tall, taller, tallest, high, higher, highest</p> <p>Low, wide, narrow, deep, shallow, thick, thin</p> <p>Far, near, close</p> <p>Metre, ruler, metre stick</p> <p>Money, coin, penny, pence, pound, price, cost, buy, sell, spend, spent, pay, change, dear(er), costs more, costs less, cheaper, costs the same as</p> <p>How much?, how many?</p> <p>Total</p>				<p>way, best way, another way</p> <p>In order, in a different order</p> <p>Not all, every, each</p>
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New maths vocabulary for year 2

Number and place value	Measure	Geometry (position and direction)	Geometry (properties of shape)	Fractions	Data/statistics	General/problem solving
Numbers to one hundred	Quarter past/to	Rotation	Size	Three quarters, one third, a third	Count, tally, sort	Predict
Hundreds	m/km, g/kg, ml/l	Clockwise, anticlockwise	Bigger, larger, smaller	Equivalence, equivalent	Vote	Describe the pattern, describe the rule
Partition, recombine	Temperature (degrees)	Straight line	Symmetrical, line of symmetry		Graph, block graph, pictogram,	Find, find all, find different
Hundred more/less		Ninety degree turn, right angle	Fold		Represent	Investigate
			Match		Group, set, list, table	
			Mirror line, reflection		Label, title	
			Pattern, repeating pattern		Most popular, most common, least popular, least common	

Calculation Policy

Addition and Subtraction

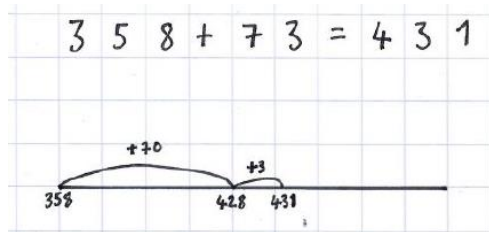
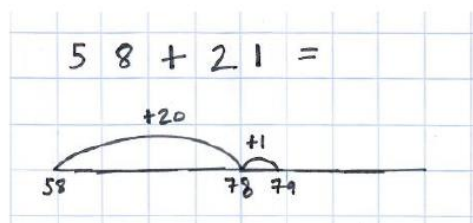
The 2014 National Curriculum outlines the progression of addition and subtraction in Key Stage 2 as introducing the column method in Year 3 with 3 digit numbers and moving towards dealing with increasing larger numbers and decimals as children progress through the Key Stage. This is outlined below:

	Year 3	Year 4	Year 5	Year 6
Formal methods expected	Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	No new content introduced for addition and subtraction. Instead, the focus is on: 'solving addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why'
Addition and Subtraction involving decimals	Add and subtract amounts of money to give change, using both £ and p in practical contexts	Solve simple measure and money problems involving fractions and decimals to two decimal places	They practise adding and subtracting decimals, including a mix of whole numbers and decimals and decimals with different numbers of decimal places	
Expectations for mental fluency in Addition and Subtraction	Add and subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens and a three-digit number and hundreds	Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency	Add and subtract numbers mentally with increasingly large numbers	They undertake mental calculations with increasingly large numbers and more complex calculations

The following pages will show the progression in the Key Stage towards the formal written methods. Pupils will soon move away from methods such as partitioning early on in Year 3 as the column method becomes more efficient. However, the table above also shows the expectation for mental fluency in addition and subtraction and it is important to remember that partitioning, rather than column methods, best represents the techniques we use for mental calculation.

Addition

Step 1: Partitioning with a number line (Year 3)



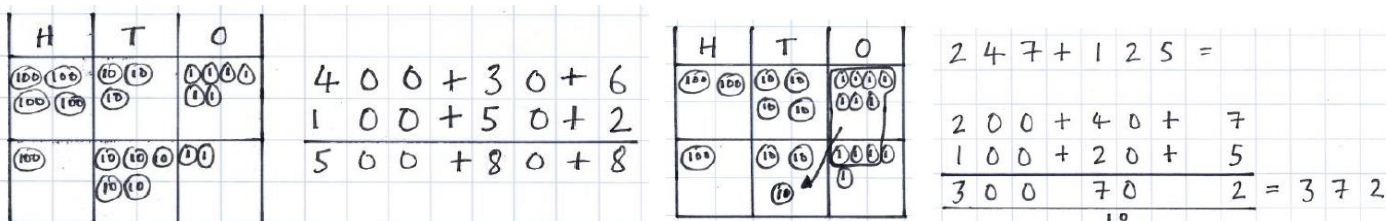
Pupils will already be familiar with this method from Year 2. The number being added is partitioned and then added to the other number in jumps. This starts with jumps of hundreds, then tens, then ones. This may also be taught with dienes cubes or place value counters.

Step 2: Partitioning without a number line (Year 3)

$$\begin{array}{r}
 85 + 37 = ? \\
 85 + 30 = 115 \\
 115 + 7 = 122
 \end{array}$$

Pupils replicate the process above in Step 1 are able to do so without the need of a diagram. Just as before, hundreds are added first, then tens, followed by the ones.

Step 3: Partitioning with expanded column method (Year 3)



This will be pupils' first exposure to adding vertically and should therefore be introduced with dienes cubes or place value counters to demonstrate the process. As seen above, the written method should accompany it at all times (the example on the right shows how to deal with crossing tens barriers).

Step 4: Expanded column method (Year 3)

$$\begin{array}{r}
 757 \\
 + 121 \\
 \hline
 8 \quad (7 + 1) \\
 70 \quad (50 + 20) \\
 800 \quad (700 + 100) \\
 \hline
 878
 \end{array}$$

This method should be used to illustrate the formal column method. It may or may not be necessary for pupils to spend time becoming fluent with it.

NB: SEE THE TABLE ON PAGE 1 FOR THE NUMBER OF DIGITS REQUIRED FOR EACH YEAR GROUP

Step 5: Formal column method without carrying (Year 3 onwards)

	6	3	2
+	1	5	4
<hr/>			
	7	8	6

It may support pupils' understanding to have the columns labelled with H, T and O when this method is first introduced. Teachers must encourage pupils to see the above as '2 ones + 4 ones equals 6 ones, 3 tens + 5 tens equals 8 tens and 6 hundreds + 1 hundred equals 7 hundreds'.

Step 6: Formal column method with carrying (Year 3 onwards)

	6	4	7	3
+	2	4	6	1
<hr/>				
	8	9	3	4
<hr/>				
		1		

The carried number is to be written below the equal sign.

Step 7: Formal column method with decimals (Year 4 onwards)

	£	6	4	.	5	0
+	£	1	9	.	6	3
<hr/>						
	£	8	4	.	1	3
<hr/>						
				,		

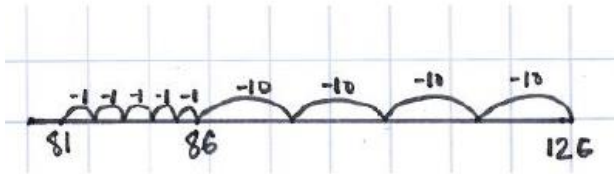
When dealing with decimals, pupils must be able to manipulate the two numbers so that they have the same number of decimal places. In the example below, the calculation was $528 + 7.49$. Therefore 528 becomes 528.00 so that the two numbers have the same number of decimal places.

$$\begin{array}{r} 528.00 \\ + 7.49 \\ \hline 535.49 \end{array}$$

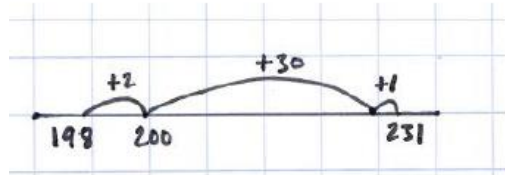
Subtraction

Step 1: Number line (Year 3)

$$126 - 45 = ?$$



$$231 - 198 = ?$$



Building on their knowledge from Year 2, pupils progress to using a number line with 3 digit numbers. They will partition the amount being subtracted to 'count back' in jumps of hundreds, tens and ones as shown in the example above left. Alternatively, if the two numbers in the calculation are close together, it may be more efficient to 'count on' to work out the difference between the two numbers, as shown in the example above right.

Step 2: Expanded column method without regrouping (Year 3)

$$98 - 35 = ?$$

	9	0	8
-	3	0	5
<hr/>			
	6	0	3

The two numbers in the calculation are partitioned into their tens and units (or hundreds, tens and units) and placed vertically. Unlike column addition, it is essential they are placed in the right order with the original total at the top and the amount being subtracted placed beneath (avoid phrases such as "The bigger number always goes at the top" as later in the Key Stage, children will find out that it is possible to subtract a bigger number from a smaller number).

Step 3: Expanded column method with regrouping (Year 3)

	6	0	
	7	0	12
-	4	0	7
<hr/>			
	2	0	5

Here, pupils are first introduced to the idea that we can regroup the number 72 into 60 and 12 for it to be possible to then subtract 40 and 7.

Step 4: Formal column method without regrouping (Year 3 onwards)

$$\begin{array}{r} 7,329 \\ - 215 \\ \hline 7114 \end{array}$$

(Year 4 example)

Pupils should already be familiar with this layout from their recent learning on column addition. Again, it may be necessary initially for pupils to label each column. As with column addition, teachers must encourage children to see the above example as '9 ones subtract 5 ones equals 4 ones, 2 tens subtract 1 ten equals 1 ten, 3 hundreds subtract 2 hundreds equals 1 hundred and 7 thousands subtract no thousands equals 7 thousands.

Step 5: Formal column method with regrouping (Year 3 onwards)

$$\begin{array}{r} \overset{2}{7}\overset{1}{3}3 \\ - 216 \\ \hline 517 \end{array}$$

$$\begin{array}{r} 932 - 457 = 475 \\ \overset{8}{9}\overset{12}{3}\overset{12}{2} \\ - 457 \\ \hline 475 \end{array}$$

Pupils are now required to regroup the number on the top in order to carry out the subtraction. Initially, pupils will have to do this once as shown in the example above left where the number 733 is regrouped as 720 and 13. Following this, children progress to being able to regroup on multiple occasions for one calculation, as demonstrated in the example above right.

Step 6: Formal column method with decimals (Year 4 onwards)

$$\begin{array}{r} 20.00 \\ - 1.42 \\ \hline 18.58 \end{array}$$

As with column addition with decimals, the first stage of column methods with decimals is ensuring both numbers have the same number of decimal places. The above example shows the calculation $20 - 1.42$ and how 20 becomes 20.00 so that both numbers have 2 decimal places.

Multiplication

Pupils beginning Key Stage 2 will already be familiar with the concept of multiplication. They will have used various techniques (such as arrays or counting in the multiple) to solve multiplication statements involving the 2, 3, 5 and 10 times tables. As outlined in the table below, the Key Stage 2 curriculum introduces the formal method of multiplication in Year 3 moving towards long multiplication in Upper Key Stage 2.

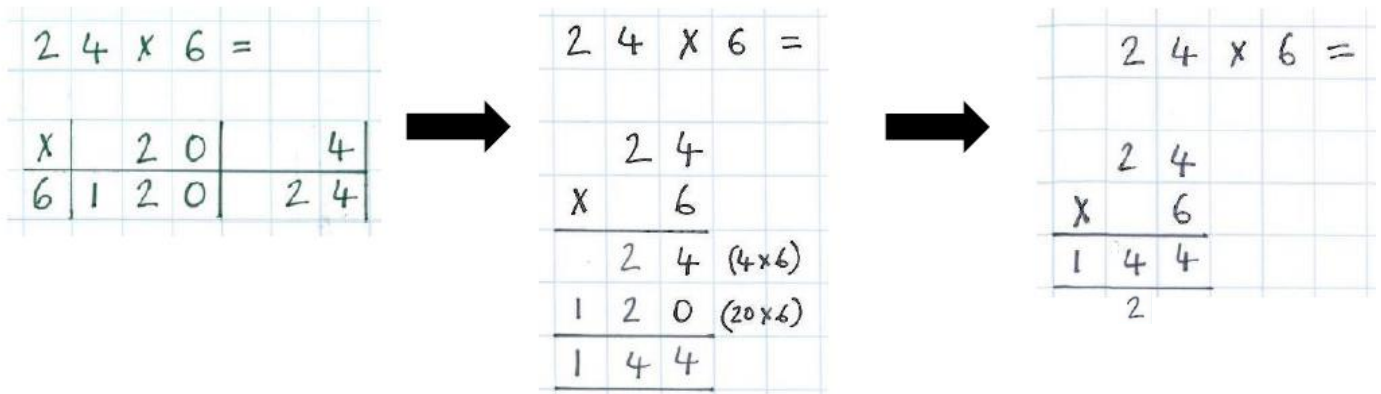
	Year 3	Year 4	Year 5	Year 6
Formal methods expected	Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
Multiplication and Division facts pupils are required to recall	2, 3, 5 and 10 (from Key Stage 1) as well as 4 and 8 times tables new for Year 3. Pupils must also be able to count in multiples of 50 and 100.	All multiplication and division facts up to the 12 times table. This includes 6, 7, 9, 11 and 12 times tables which are new to Year 4. Pupils must also be able to count in multiples of 25 and 1000.	Children should already be fluent in all times tables up to the 12 times table. In Year 5, they should be able to count in powers of 10 (10, 100, 1000, 10,000, 100,000 and 1,000,000) starting on any number less than 1,000,000.	

Step 1: The grid method using equipment, arrays and then just the numbers themselves (Year 3)

The diagram illustrates the progression of the grid method for multiplication. It starts with the equation $18 \times 3 = 54$. The first step shows a grid where 18 is split into 10 and 8, and 3 is multiplied by each to get 30 and 24, which are then added to get 54. The second step shows the same grid but with dots instead of numbers. The third step shows the final numerical grid method.

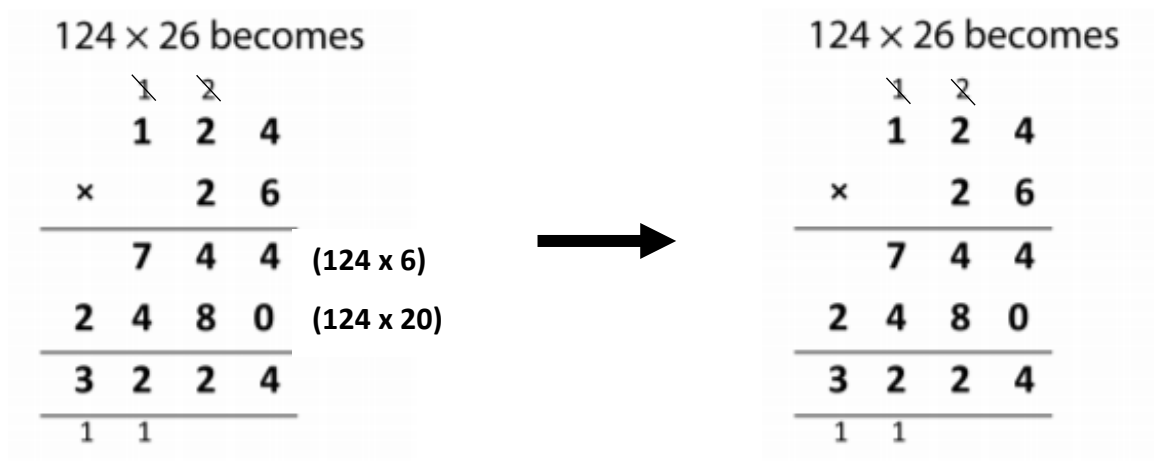
Here, the number that is being multiplied (18 in this case) is partitioned into tens and ones. This breaks up the calculation into a pair of multiplication sentences (3×10 and 3×8) which the child can work out and then add together. This should be introduced with equipment (such as place value counters or dienes cubes) then reinforced with pictorial representations (such as arrays) before relying solely on the numbers.

Step 2: Grid method to Short Multiplication (Year 3 onwards)



This step is so that pupils are able to understand how the method of short multiplication works. The grid method is transferred into an expanded vertical column and then this develops into the formal written method or short multiplication (2 digit multiplied by 1 digit in Year 3 followed by 3 digit multiplied by 1 digit in Year 4).

Step 3: Long multiplication (Year 5 and 6)



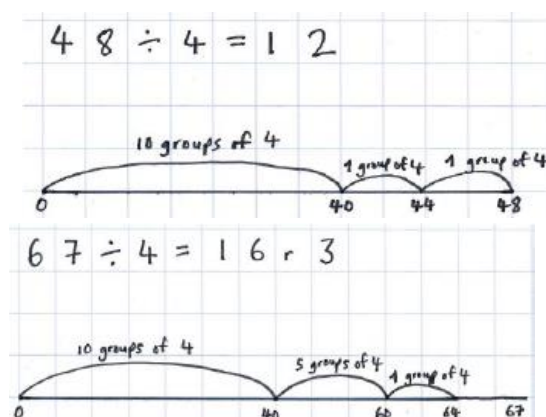
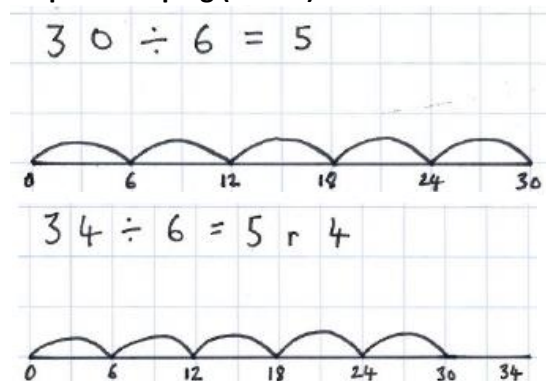
Long multiplication is introduced first with brackets describing each calculation to reinforce the procedure of doing 124 multiplied by 6 ones followed by 124 multiplied by 2 tens (20). This can be removed once this is internalised. Also, note how the carried values are crossed out once they have been used to avoid confusion when there are multiple values being carried.

Division

Pupils beginning Key Stage 2 will already be familiar with the concept of division. They will have used various techniques (such as counting in the multiple) to solve division statements involving the 2, 3, 5 and 10 times tables. As outlined in the table below, the Key Stage 2 curriculum introduces the formal method of division in Year 3 and Year 4 (although this only becomes statutory in Year 5) moving towards long multiplication in Upper Key Stage 2.

	Year 3	Year 4	Year 5	Year 6
Formal methods expected	Short division with TU divided by U giving exact answers (non-statutory) Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know.	Short division with HTU divided by U giving exact answers (non-statutory)	Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context, either as fractions, decimals, rounding to the nearest integer or leaving it as a remainder.	Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division (or short division where appropriate), and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
Multiplication and Division facts pupils are required to recall	2, 3, 5 and 10 (from Key Stage 1) as well as 4 and 8 times tables new for Year 3. Use division facts (for example, using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts (for example, $30 \times 2 = 60$, $60 \div 3 = 20$ and $20 = 60 \div 3$).	All multiplication and division facts up to the 12 times table. This includes 6, 7, 9, 11 and 12 times tables which are new to Year 4. Pupils practise mental methods and extend this to three-digit numbers to derive facts, (for example $600 \div 3 = 200$ can be derived from $2 \times 3 = 6$).	Continued use of multiplication and division facts up to the 12 times table.	Continued use of multiplication and division facts up to the 12 times table.

Step 1: Grouping (Year 3)



Pupils will already be familiar from Year 2 with the concept of counting groups of the multiple to reach the total for division problems. The number line formalises this and shows how to handle remainders (see the two examples above with remainders). This progresses towards a more efficient method (as shown on the right hand examples) where larger groupings are made.

Step 2: Short division (Year 3 and Year 4 non-statutory, Year 5 onwards statutory)

(Year 3 and Year 4)

In this introduction to Short Division, there is no carrying and there are no remainders. Emphasis is made on asking the question: 'how many groups of 3 are in 90?' rather than the procedural '3s into 9 go 3'. Answers are whole numbers.

9	6	÷	3	
	3	2		
3	9	6		

Here, remainders are carried over to the following column. Again, emphasis is made on the place value at each stage: "We can make 1 hundred groups of 4 in 500, with 1 hundred remaining which carries over to the tens column" etc. Again, answers are whole numbers.

5	4	8	÷	4	
	1	3	7		
4	5	4	2	8	

(Year 5 and Year 6)

In Year 5, pupils are expected to divide a 4 digit number by a one digit number. They are also expected to deal with remainders appropriately, deciding whether it should be left as a remainder, turned into a fraction or a decimal or rounded to the nearest whole number.

Left as a remainder

In this example, the remainder is simply left as a remainder using 'r' to indicate this.

7	9	4	8	÷	5	
	1	5	8	9	r3	
5	7	2	9	4	4	8

Displayed as a fraction

In this example, we take the 3 that is left over as a remainder and divide it by 5 making the fraction $\frac{3}{5}$.

7	9	4	8	÷	5	
	1	5	8	9	$\frac{3}{5}$	
5	7	2	9	4	4	8

Displayed as a decimal

In this example, the number being divided (7948) is given a decimal value of 0 (7948.0) so that the remaining 3 can be carried over. Notice the decimal point written above and below the bus stop in the same position.

7	9	4	8	÷	5	
	1	5	8	9	.	6
5	7	2	9	4	8	.0

Rounded to the nearest integer (whole number)

In some cases (such as 'How many pencil cases would you need to fit 8,396 pencil crayons if each one could hold a maximum of 9 pencil crayons?'), the remainder is rounded up to the nearest whole number. In other cases (such as 'How many complete pencil cases would you be able to make?') the answer is rounded down to the previous integer.

8	3	9	6	÷	9				
	0	9	3	2	r8	→	9	3	3
9	8	3	2	6					

Fractions

By the end of Key Stage 1, pupils should be familiar with the concept of fractions – dealing with halves, thirds and quarters – and will even be used to calculating fractions of amount, both for unit fractions (such as $\frac{1}{4}$ of 12) and for non-unit fractions ($\frac{3}{4}$ of 20). Key Stage 2 builds on this foundation. By the end of the Key Stage, pupils should be able to add, subtract, multiply and divide fractions in their different forms (unit and non-unit as well as improper fractions and mixed numbers). The table below shows this progression for each year.

	Year 3	Year 4	Year 5	Year 6
Calculation work with fractions	<p>Add and subtract fractions with the same denominator within a whole.</p> <p>Calculate fractions of amount with unit and non-unit fractions with small denominators.</p>	<p>Add and subtract fractions with the same denominator beyond a whole.</p> <p>Calculate fractions of amount with increasingly harder fractions including non-unit fractions where the answer is a whole number.</p>	<p>Add and subtract fractions with the same denominator and denominators that are multiples of the same number.</p> <p>Use mixed numbers and improper fractions to write mathematical statements beyond 1.</p> <p>Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams</p>	<p>Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.</p> <p>Multiply simple pairs of proper fractions, writing the answer in its simplest form.</p> <p>Divide proper fractions by whole numbers</p>

Addition and Subtraction of Fractions

Step 1: Addition and Subtraction of Fractions with the same denominator (Year 3 and Year 4)

At this stage, children need to be familiar with the concept that the numerators are added or subtracted but the denominators remain the same.

$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$

$$\frac{7}{8} - \frac{5}{8} = \frac{2}{8}$$

Year 3 example: Add and subtract fractions with the same denominator within a whole.

$$\frac{5}{9} + \frac{6}{9} = \frac{11}{9}$$

$$\frac{7}{5} - \frac{3}{5} = \frac{4}{5}$$

Year 4 example: Add and subtract fractions with the same denominator beyond a whole.

$$\frac{6}{7} + \frac{2}{7} = \frac{8}{7} = 1 \frac{1}{7}$$

Year 5 example: Add and subtract fractions with the same denominator beyond a whole but converting improper fraction to a mixed number.

Step 2: Addition and Subtraction of Fractions with denominators that are multiples of the same number (Year 5)

This is when children are first introduced to the idea that they can manipulate two unlike fractions so that they can be added or subtracted (or ordered and compared). At this stage, one fraction is written in its equivalent form so that the denominators are the same. Pupils must get used to rewriting the calculation underneath.

$$\frac{3}{8} + \frac{1}{2} = \frac{3}{8} + \frac{4}{8} = \frac{7}{8}$$

$$\frac{2}{3} + \frac{1}{9} = \frac{6}{9} + \frac{1}{9} = \frac{5}{9}$$

Step 3: Addition and Subtraction of Fractions with different denominators using equivalent fractions (Year 6)

In Year 6, children must be able to manipulate both fractions so that their denominators are the same in order for them to be added or subtracted (or ordered or compared). In some circumstances, the lowest common denominator is found by multiplying the two denominators together (as shown in the addition example here) however there may be times when there is an alternative common multiple as shown in the subtraction example. In any case, the number sentence is rewritten underneath as illustrated in the examples.

Handwritten calculation showing the addition of $\frac{3}{5} + \frac{1}{3}$. The denominators 5 and 3 are circled, with arrows pointing to a common denominator of 15. The fractions are converted to $\frac{9}{15} + \frac{5}{15} = \frac{14}{15}$. Multiplication factors $\times 3$ and $\times 5$ are indicated.

Handwritten calculation showing the subtraction of $\frac{1}{3}$ from $\frac{5}{6}$. The denominators 6 and 3 are circled, with arrows pointing to a common denominator of 12. The fractions are converted to $\frac{10}{12} - \frac{4}{12} = \frac{6}{12}$, which is simplified to $\frac{1}{2}$. Multiplication factors $\times 2$ and $\times 3$ are indicated.

Step 4: Addition and Subtraction of Mixed Numbers with different denominators (Year 6)

Also in Year 6, children must be able to calculate with mixed numbers. As shown in this example, the mixed numbers are converted to improper fractions and then the calculation can be carried out as shown in Step 3 by finding a common denominator (6 in this case). The improper fraction can always be converted back to a mixed number at the end.

NB. In this example, it may have been more efficient to add the two integers ($2 + 1 = 3$) and then add the two fractions ($\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$) and finally put them together to make $3\frac{5}{6}$ but problems occur if this were a subtraction as $2 - 1 = 1$ but then $\frac{1}{3} - \frac{1}{2}$ would create a negative number.

Handwritten calculation showing the addition of mixed numbers $2\frac{1}{3} + 1\frac{1}{2}$. The mixed numbers are converted to improper fractions $\frac{7}{3} + \frac{3}{2}$. A common denominator of 6 is found, resulting in $\frac{14}{6} + \frac{9}{6} = \frac{23}{6}$.

Multiplication of Fractions (Year 6)

Proper fractions multiplied by a proper fraction

Very simply, the numerators are multiplied together and the denominators are divided together. Sometimes the answer may need to be simplified as shown in this example.

Handwritten calculation showing the multiplication of $\frac{2}{3} \times \frac{5}{6} = \frac{10}{18} = \frac{5}{9}$.

Fractions multiplied by a whole number

When dealing with whole numbers in calculations involving fractions, we convert the whole number into an improper fraction by putting it as the numerator over 1 as a denominator. After this has been done, the calculation becomes identical to the proper fraction multiplied by a proper fraction example above.

Handwritten calculation showing the multiplication of $\frac{4}{5} \times 3 = \frac{4}{5} \times \frac{3}{1} = \frac{12}{5}$.

The only further complexity arises when it is a mixed number multiplied by a whole number. In this scenario, the mixed number is converted to an improper fraction as shown in the example on the right.

Handwritten calculation showing the multiplication of a mixed number $1\frac{3}{4} \times 3 = \frac{7}{4} \times \frac{3}{1} = \frac{21}{4} = 5\frac{1}{4}$.

Division of Fractions

Division of proper fractions by whole numbers (Year 6)

$$\frac{2}{5} \div 3 = \frac{2}{5} \div \frac{3}{1} = \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$$

As with multiplication, when dealing with whole numbers in a fraction calculation, the first step is to convert the whole number into an improper fraction by putting it as the numerator over 1 as a denominator. Once this has been done, pupils follow the 'KFC' procedure: kee the first fraction the same, flip the second fraction and change the symbol from a multiplication to a divide. Once this has been done, the calculation becomes a simple proper fraction multiplied by a proper fraction.

Calculating Fractions of Amount (Year 3 onwards)

As outlined in the table progression table for fractions above, children have to be able to find fractions of amount throughout Key Stage 2. The example below shows how these calculations are to be set out. This format will be identical from Year 3 to Year 6; the only thing that may differ is the complexity of the numbers involved.

$$\begin{array}{l} \frac{3}{7} \text{ of } 21 \\ \frac{1}{7} \text{ of } 21 = 21 \div 7 = 3 \\ \frac{3}{7} \text{ of } 21 = 3 \times 3 = 9 \end{array}$$

Calculating Percentages of Amount (Year 6)

As calculating with percentages makes up a relatively small part of the National Curriculum compared with fractions, the layout for working out percentages of amount is included in the fractions section on this document. As you can see from the example, it follows a similar layout to calculating fractions of amount.

$$\begin{array}{l} 40\% \text{ of } 720 \\ 10\% = 72 \\ \times 4 \quad 40\% = 288 \times 4 \end{array}$$

New maths vocabulary for year 3

Number and place value	Addition and subtraction	Multiplication and division	Measure	Geometry (position and direction)	Geometry (properties of shape)	Fractions	Data/statistics
Numbers to one thousand	Column addition and subtraction	Product Multiples of four, eight, fifty and one hundred Scale up	Leap year Twelve-hour/twenty-four-hour clock Roman numerals I to XIII	Greater/less than ninety degrees Orientation (same orientation, different orientation)	Horizontal, vertical, perpendicular and parallel lines	Numerator, denominator Unit fraction, non-unit fraction Compare and order Tenths	Chart, bar chart, frequency table, Carroll diagram, Venn diagram Axis, axes Diagram

New maths vocabulary for year 4

Number and place value	Multiplication and division	Measure	Geometry (position and direction)	Geometry (properties of shape)	Fractions and decimals	Data/statistics
Tenths, hundredths Decimal (places) Round (to nearest) Thousand more/less than Negative integers Count through zero Roman numerals (I to C)	Multiplication facts (up to 12x12) Division facts Inverse Derive	Convert	Coordinates Translation Quadrant x-axis, y-axis Perimeter and area	Quadrilaterals Triangles Right angle, acute and obtuse angles	Equivalent decimals and fractions	Continuous data Line graph

New maths vocabulary for year 5

Number and place value	Addition and subtraction	Multiplication and division	Measure	Geometry (position and direction)	Geometry (properties of shape)	Fractions, decimals and percentages
Powers of 10	Efficient written method	Factor pairs Composite numbers, prime number, prime factors, square number, cubed number Formal written method	Volume Imperial units, metric units	Reflex angle Dimensions	Regular and irregular Polygons	Proper fractions, improper fractions, mixed numbers Percentage Half, quarter, fifth, two fifths, four fifths Ratio, proportion

New maths vocabulary for year 6

Number and place value	Addition and subtraction	Multiplication and division	Geometry (position and direction)	Geometry (properties of shape)	Fractions, decimals and percentages	Algebra	Data/statistics
Numbers to ten million	Order of operations	Order of operations Common factors, common multiples	Four quadrants (for coordinates)	Vertically opposite (angles) Circumference, radius, diameter	Degree of accuracy Simplify	Linear number sequence Substitute Variables Symbol Known values	Mean Pie chart Construct